

Hilbert function spaces and Pick's interpolation theorem

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Pick's interpolation theorem is a century old theorem in complex analysis regarding interpolation with bounded analytic functions. While originally a purely function theoretic result, this theorem is intimately connected with Hilbert spaces and operator theory. Indeed, Pick's theorem can be regarded as a statement about multiplication operators on the Hardy space H^2 , one of the best understood spaces of holomorphic functions.

Pick's theorem holds in other Hilbert function spaces, such as the Dirichlet space and the Drury–Arveson space. This observation provides a functional analytic approach to function theoretic questions in these spaces. Over the last 30 years, it turned out that many theorems, which were initially thought to be specific to the Hardy space, actually hold in these more general spaces.

I will discuss the following topics:

- Pick's theorem and its operator theoretic interpretation,
- complete Pick spaces,
- interpolating sequences,
- weak products,
- factorization.

Part of these lectures is based on joint work with Alexandru Aleman, John M^cCarthy and Stefan Richter.